 February, 1935

Appalachian Forest Experiment Station

Computations of National Forest Timber Survey Data

Curve Fitting and Volume Table Checks and Revisions

The purpose of this technical note is to outline procedures particularly adapted to the computation of the present timber surveys on National Forests. The responsibility for much of this work has of necessity been placed upon Junior Foresters, many of whom are for the first time faced with the assembling and computation of large quantities of management plan data. It is to these men that the following discussions are primarily directed. In this and forthcoming Technical Notes the various procedures dealt with were chosen as important after conferences with forest officers engaged in the direction of timber surveys and management plan work.

The immediate need is for volume tables which can be used with confidence on the different forests. Because the checking and revision of volume tables involves considerable use of free hand curves, the first section of this note will be devoted to practical suggestions for the proper fitting of such curves. The second section will outline in detail the various steps for checking and, if necessary, revising existing regional tables for local use on the national forests.

1 - Curve Fitting

The curve of merchantable height over d.b.h. is probably the first curve with which the timber survey computer comes in contact. For this reason such a curve will be used as an illustration of curve fitting. Under the present system for each species a merchantable height over d.b.h. curve is drawn for each of the 3 site classes. In preparation for plotting on cross-section paper, computations of sample trees should be summarized as in Table 1. The average merchantable height (column 3), of each inch class should then be plotted over the actual average d.b.h. of the diameter class as shown in column 2. Plotted points representing an average must always be accompanied on the cross section paper by the number of items (column 4 of Table 1) upon which the average is based (See the small figures near each point on Fig. 1) After plotting the points, draw a smooth curve through them by inspection. This curve is probably not an accurate expression of the data, but furnishes a basis to work from in revising the position and shape of the curve. This revision is accomplished by the following steps:

1. Count the number of small squares between the first plotted point and the curve. The counting must be done vertically. (For the first point on Fig. 1 the number of squares is 1.5).

2. Multiply the number of squares by the number of items (in this case number of trees) which were used to determine the average of the particular class represented by the point. (On Fig. 1 1.5 x 3 = 4.5)

3. If the point is above the curve, record the product derived in step 2 under a plus sign on the sheet (See Fig. 1) If the point is be-

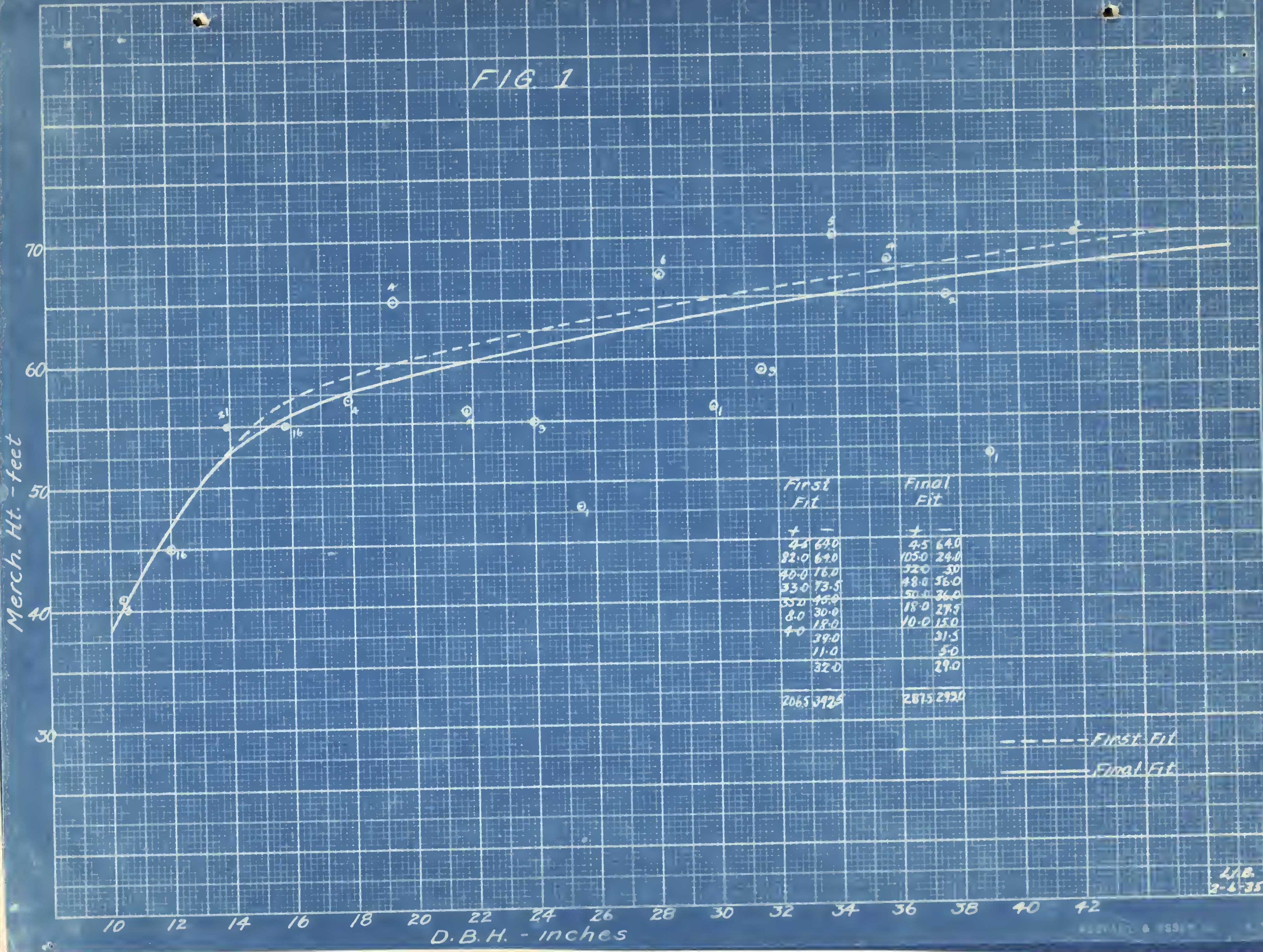

Merch. Ht. - feet
30
40
50
60
70
D.B.H. - inches
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
First Fit
+ −
4.5 64.0
82.0 64.0
40.0 16.0
33.0 73.5
55.0 46.0
8.0 30.0
4.0 18.0
39.0
11.0
32.0
206.5 342.5
Final Fit
+ −
4.5 64.0
105.0 24.0
32.0 3.0
48.0 56.0
50.0 36.0
18.0 27.5
10.0 15.0
31.5
5.0
29.0
287.5 292.0
First Fit
Final Fit
L.R.B.
2-6-35

FIG. 1

Table I

Site I - Yellow Poplar

Merchantable Heights.

1 DBH Class	2 Average DBH	3 Av. Merchant- able Height	4 Number of trees
10	10.4	41	3
12	12.0	45	16
14	13.9	55	21
16	15.8	55	16
18	17.9	57	4
20	19.4	65	4
22	21.8	56	7
24	24.0	55	3
26	25.5	48	1
28	28.2	67	6
30	29.9	56	1
32	31.5	59	3
34	33.9	70	5
36	35.7	68	4
38	37.6	65	2
40	39.0	52	1
42	41.9	70	2

low the curve, record it under a minus sign.

4. Proceed in this manner with each plotted point, recording the products derived from points above the curve under a plus sign, and those below the curve under a minus sign.

5. When all products have been worked out, compare the sum of the plus products with the sum of the minus products. If the curve is an adequate expression of the data, these sums should be approximately equal.

In Figure 1 the broken line represents the first curve drawn through the plotted points. It is obvious from the sums of the plus and minus products shown in Figure 1 under the heading "First Fit" that the minus products exceed the plus products by 186 (392.5 - 206.5) This indicates that there is too much weight below the curve and that it must be lowered to give a better expression of the data. The curve was then lowered as indicated by the solid line and its shape changed slightly and new products computed. Under the heading "Final Fit" of Fig. 1 it is apparent that the sum of the plus products is approximately equal to the sum of the minus products. The difference between these two values should not exceed 1 percent of their sum if the curve is properly fitted. In this case the fit is not highly satisfactory since the difference between the sums of the plus and minus products is 5.5 (293.0 - 287.5) while 1% of their total is 5.8 (1% of 293.0 + 287.5)

A further and equally important test of the adequacy of the final curve is obtained by comparing the sums of all products regardless of their algebraic sign, with a similar sum for other trial fits. For example, the sum of the products of the first fit is 599 (206.5 + 392.5) while the corresponding sum of the final fit is 580.5 (287.5 + 293.0) a lower figure than for the first fit. Since these sums are a measure of the total scatter of the data around the curve, the lower figure (580.5) shows a closer correlation between the final curve and the plotted points than did the first curve. The mere balancing of the plus and minus products is not then, the final test of any curve, since a very inadequate curve may show these products to balance. For instance, in Figure 1 a straight horizontal line could have been fitted to the plotted points in such a position that the sum of the plus and the sum of the minus products were equal. However, the figure obtained by adding these two sums would greatly exceed the similar figure for the final fit (580.5). Therefore, a curve is not a correct expression of the data until the sum of the plus and minus products balance and until the total of these plus and minus products is the lowest possible figure. However, this last statement must not be interpreted as a recommendation for S-shaped curves or curves of a wave like character in which several crests and troughs are shown. Past experience has proved that the simple correlations such as presented here are most perfectly expressed by smooth curves without irregularities. Plotted points indicating S-shaped or other irregular curves are usually the result of inadequate basic data.

II. Volume Table Checks and Revisions

The revision of regional volume tables for local use consists basically of comparing the volumes of locally cut trees with the volumes of these same trees as derived from the tables for the particular species in question. In case the table does not fit the local data, the scaled volumes of the locally cut trees are used to correct the table. The basic

method of checking volume tables for local use was originally suggested by Donald Bruce.

Unless other tables have been used successfully it is suggested that the latest revision of the acquisition volume tables as given on the large form "Timber Estimate Computation Sheet for Southern Appalachians" be used for checking and revision.

In making volume table checks it is essential that the type of dimensions measured in trees cut on local logging jobs be the same as the dimensions used in the table to be checked. For instance, in checking a table where the volumes are given by DBH and log lengths, measurements in the field should include d.b.h. and the dimensions of each log. If the volume table to be checked is based on d.b.h. and total height, the trees scaled in the field must be measured for total height and a record of the number and sizes of logs would not be necessary provided the gross scale and volume of defect is recorded. Gross scale should be recorded as the total log scale including defect of the tree as cut in the woods. The practice, common in Appalachian hardwoods, of cutting log lengths in multiples of 2 feet invariably results in more logs being cut in lengths under 16 feet than over. This practice results in a higher gross log scale than if the trees had been cut in even 16 foot and 8 foot lengths. Volume tables in which merchantable heights are determined by the number of 16 foot logs will then be low if the trees from which the volume table was built were scaled as 16 foot and 8 foot logs. Hence in order to reduce one source of error in computing timber estimate data the volume table should show the actual gross log scale of trees as cut on local operations.

As an example, part of a volume table for yellow poplar (Table 51, Vol. Table for Eastern Hardwoods) will be checked and revised from measurements of 63 trees cut near Parsons, West Virginia.

In order to make the check it is first necessary to have at hand the volume table to be checked and a tabulation showing for each tree measured locally its gross scale, d.b.h. to 0.1 inch and number of 16 foot logs and fractions thereof.

The following steps are than necessary:

1. Prepare a tabulation such as Table II. Columns 1, 2, 3 and 5 will be filled in first from the data at hand. In column 3 the fractions of 16 foot logs result from converting the cuts actually made in the tree ingo 16 foot logs. Although not actually necessary for this particular step, the recording together of all trees in an inch class will save time in a later operation.

2. Enter in column 4 the volume of each tree as determined by the table to be checked. It will be necessary to interpolate in the volume table for fractional log lengths and for d.b.h's to tenths of an inch. A simple way of fixing this interpolation method in the mind is to proceed in the following manner using tree number 2 in Table 2 as an example:

(a) Inspection of the volume table (Table IV) shows that the estimated volume of tree No. 2 falls somewhere between the volumes of 10 and 12 inch trees and also between the volumes of 2 and 3 log trees of these diameters. Record these values on a piece of scratch paper as follows:

TABLE NO.II
Yellow Poplar

Correction of Volume Tables for Local Use

1	2	3	4	5	6	7
			Volume in bd.ft.			
Tree No.	DBH In.	No. logs	from table	local trees	Diff.	Diff. in %
1	10.0	2	50	60	10	20.0
2	10.4	2½	71	70	1	1.4
3	10.8	3¼	106	90	16	15.1
4	11.0	3	100	110	10	10.0
5	11.0	2¾	90	70	20	22.2
6	11.0	3	100	90	10	10.0
7	11.2	2 7/8	99	80	19	19.2
8	11.7	2	67	70	3	4.5
9	11.8	3¼	130	180	50	38.5
10	11.9	2	69	70	1	1.4
11	12.0	2½	95	80	15	15.8
12	12.1	2 5/8	103	100	3	2.9
13	12.2	3¼	140	110	30	21.4
14	12.4	2	74	100	26	35.1
15.	12.4	3 5/8	169	140	29	17.2
16	12.6	3 7/8	190	140	50	26.3
17	12.6	3½	166	140	26	15.6
18	12.7	2¼	92	100	8	8.7
19	12.8	2¼	94	75	19	20.2
20	13.0	3½	178	130	48	26.9
21	13.0	3½	178	140	38	21.3
22	13.0	3 5/8	186	150	36	19.3
23	13.2	4 1/8	224	210	14	6.2
24	13.2	3	150	120	30	20.0
25	13.3	3¼	169	120	49	29.0
26	13.4	2 5/8	128	120	8	6.2
27	13.6	3 5/8	202	180	22	10.9
28	13.6	3 7/8	220	190	30	13.6
29	13.7	3 3/8	188	150	38	20.2
30	13.7	3	162	110	52	32.1
31	13.8	2¼	107	130	23	21.5
32	14.0	2¾	150	110	40	26.6
33	14.2	3	175	140	35	20.0
34	14.4	3 1/8	189	180	9	4.8
35	14.4	4	254	220	34	13.4
36	14.6	4 1/8	270	210	60	22.2
37	14.6	4	261	160	101	38.7
38	14.7	3 7/8	255	360	105	41.2
39	14.8	3 ¼	210	160	50	23.8
40	14.9	4 1/8	280	180	100	35.7
41	15.0	3 ¾	255	240	15	5.9
42	15.0	2 5/8	161	160	1	0.6
43	15.1	3 ½	238	190	48	20.2
44	15.2	4 ¼	300	240	60	20.0
45	15.4	3 5/8	258	200	58	22.5
46	15.4	3 3/8	236	200	36	15.2
47	15.6	3 ¾	274	240	34	12.4
48	15.6	4 3/8	322	260	62	19.2
49	15.6	2 7/8	198	220	22	11.1
50	15.7	2 3/8	152	120	32	21.0
51	15.9	3 ¼	240	270	30	12.5
52	16.6	3 7/8	322	230	92	28.5
53	16.6	2 3/8	169	170	1	0.6
54	16.6	4 3/8	361	340	21	5.8
55	16.7	3 ¾	313	260	53	16.9
56	16.7	3	238	180	58	24.4
57	17.1	4 ¼	373	290	83	22.2
58	17.1	3	248	270	22	8.9
59	18.6	3 ¾	385	310	75	19.5
60	18.7	3 ¼	324	260	64	19.7
61	19.0	4 ¾	514	240	274	53.3
62	19.1	4 ¼	466	280	186	40.0
63	21.3	3	369	330	39	10.6
			12857	10845		1170.1

$$\begin{array}{r} 12857 \\ -10845 \\ \hline 2012 \end{array}$$

$\frac{*201200}{12857}$ = $\pm$15.6% - Aggregate Difference in percent

Table is 15.6% high in aggregate.

$\frac{1170.1}{63} = \pm 18.6\%$ - Average individual deviation in %

* Two zeros are added for computation of percentage.

	2 log	3 log
10 in.	50	80
12 in.	70	120

The tree for which the volume table value is to be computed has a d.b.h. of 10.4 and contains $2\frac{1}{2}$ logs. The next step is to determine the volume table value of 2 and 3 log trees which are 10.4 inches in diameter. There is a spread of 20 tenth inches between 10.0 and 12.0 so that a 2 log tree 10.4 inches in d.b.h. will have a volume equal to the volume of a 10.0 inch, 2 log tree plus $\frac{4}{20}$ of the difference in volume between a 10.0 inch 2 log tree and a 12.0 inch 2 log tree. In this case the computation is $\frac{4}{20}$ x (70 - 50) = 4. A 10.4 inch 2 log tree then has an estimated volume of 50 + 4 or 54 board feet. A similar computation for 3 log trees shows the volume of a 10.4 inch, 3 log tree to be 88 feet, $\frac{4}{20}$ x (120 - 80) + 80 = 88.

The computation then appears as follows:

	2 log	$2\frac{1}{2}$ log	3 log
10	50		80
10.4	54		88
12	70		120

Continuing the computation, a 10.4 inch, $2\frac{1}{2}$ log tree then has an estimated volume of 54 + 1/2 (88 - 54) = 71 board feet which is the volume recorded in column 4.

This interpolative method appears laborious, but failure to follow it will result in errors in columns 6 and 7 which are unfair to the table and which will result in showing larger measures of error (see later discussion of Aggregate Difference and Average Individual Deviation) than actually exist in the table.

3. Obtain totals for columns 4 and 5. The difference between the two totals expressed as a percentage indicates the aggregate error of the table and whether it is high or low, a plus sign before the percentage indicating the table is high, and vice versa. If the totals of the volumes estimated from the volume table (col. 4) exceed the total volume of local trees (col. 5) the volume table will give results in excess of actual values and vice versa. Note the computation of Aggregate Difference at the end of Table II. The Aggregate Difference measures the total amount of error in the volume table, or stated differently, it measures the amount of error which will result from use of the table in computing volumes based in a timber cruise provided the diameters measured in the cruise contain the entire range of diameters present in the volume table.

Another and equally important measure of accuracy must be made. This is accomplished by the following steps:

4. With column 4 filled in, obtain the individual differences of the values in columns 4 and 5 and enter in column 6. Column 6 then merely lists the differences in board foot volume between actual local scale and computed volume table values for trees of the same dimensions.

5. Using columns 4 and 6, compute the individual percentages of error found in the table and enter these percentages in column 7.

6. Total the values in column 7 and divide by the number of trees

used in the test to obtain the Average Individual Deviation.

As already pointed out, the aggregate difference of +15.6% in this particular case indicates that the table gives much higher values for the local trees than actually exist. It is entirely possible for the aggregate difference to be only 1% and for the table to still be quite inaccurate. This is due to the fact that a table may give very high values for small trees and low values for large trees which compensate to give a very low aggregate difference. If such a table is applied to pole stands where the majority of the saw log timber is small, the table may be considerably in error. This is also true where only a few trees of a given species are present in a stand. It is under such conditions that the Average Individual Deviation comes into the picture. This measure of accuracy indicates the possible amount of error which may be expected in case volumes are computed for only a limited number of trees or diameter classes.

In general a volume table should have an aggregate difference of not over 2% and the average individual deviation should not be greatly in excess of ± 15%.

After completing step 3, if the Aggregate Difference is in excess of 2% showing that the volume table needs correction, the computation of the Average Individual Deviation (Step 4, 5 and 6) may be postponed until later, and step 7 begun immediately. In cases where the Aggregate Difference is high the Average Individual Deviation means little or nothing and time will be saved by postponing its computation until the volume table is revised. In the case presented in Table II it is obvious that the volume table needs correction.

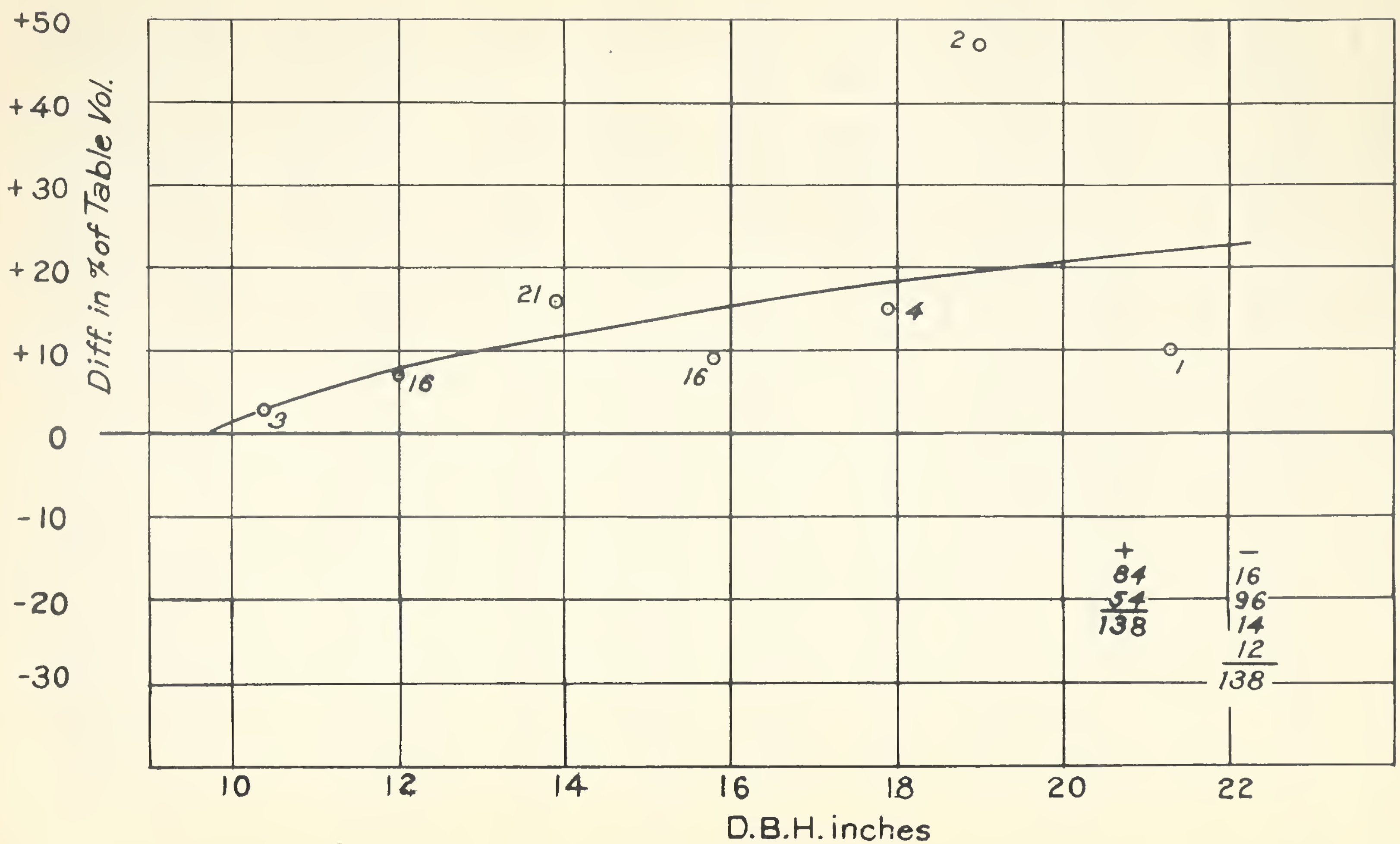
The next steps are as follows:

7. For each d.b.h. class obtain totals for columns 4 and 5 (Table II) and enter these totals in columns 3 and 4 of Table III. Also determine the average d.b.h. of each diameter class and record in column 2.

Table III
Yellow Poplar

1 DBH Class	2 Average DBH	3 Total Volume from table	4 Total vol.scale local trees	5 Diff.in bd. ft.	6 Diff.in % of table volume	7 Number of Trees
10	10.4	227	220	+7	+3.1	3
12	12.0	1778	1655	+123	+6.9	16
14	13.9	4136	3470	+666	+16.1	21
16	15.8	3876	3520	+356	+ 9.2	16
18	17.9	1330	1130	+200	+15.0	4
20	19.0	980	520	+460	+47.0	2
22	21.3	369	330	+ 39	+10.6	1
						63

FIG. 2



+50
+40
+30
+20
+10
0
-10
-20
-30
Diff. in % of Table Vol.
3
16
21
16
4
2
1
+
84
54
138
—
16
96
14
12
138
10
12
14
16
18
20
22
D.B.H. inches

8. Compute the differences between columns 3 and 4 and enter the differences in column 5. The percentage which each difference is of the volume table value is then computed and entered in column 6. In this case algebraic signs are used to indicate whether the differences are high or low, a plus sign indicating the table gave values above the local scale and a minus sign indicating the table was lower than local scale.

9. The percentages of column 6, Table III are then plotted as in Fig. 2, care being taken to plot minus percentages below the zero line and plus percentages above it. In this case all percentages are plus.

10. A curve is then carefully fitted through these points (See Section 1)

11. For each inch class the smoothed percentage correction for the volume table are read off and entered in column 7 of Table IV. Since that portion of the curve below the zero line indicates the d.b.h. classes for which the volume table gives low values, the tabular values must be raised by the percentages read from the curve and vice versa. Therefore, plus percentages indicate that the table is too high and its values must be lowered. Minus percentages indicate that the table is too low and its values must be raised. In this case all volumes in the table must be lowered.

In Table IV is presented the original volume table and in Table V the new table is shown after being corrected by applying to Table IV the percentages read from Figure 2.

There remains then the final check of the revised volume table. This requires a repetition of steps 1-6 inclusive, using the revised volume table. The actual check is presented in Table VI.

Comparison of Tables I and V indicate that considerable improvement in the volume table has resulted from the revision. The aggregate difference has been reduced from +15.6% to +1.9% and the average individual deviation from ± 18.6% to ± 15.2%.

The revised volume table can now be considered accurate within its limits of error insofar as the local trees are representative of the forest in which the table is to be used.

At first glance the procedure outlined for revision of regional volume tables will doubtless appear somewhat cumbersome and time consuming. However, it requires fewer trees and less time than the construction of new tables. In the example already given about 9 man hours were required by an experienced man to accomplish the computing necessary for the original check, revision, and final check.

The number of trees required for an adequate volume table check and revision cannot be stated in other than general terms. Species such as the conifers and yellow poplar have a single central stem and the merchantable top diameter and merchantable length of stem is subject to less variation than is the case with the oaks and other hardwoods. Consequently fewer trees of these species will be necessary for an adequate check than is the case in species with spreading, many-forked crowns. Approximately 50 trees for the single stemmed species and from 75 to 100 trees for other species

Table IV
Yellow Poplar
Original Volume Table
to be checked

1	2	3	4	5	6	7
	Number of 16 foot logs					
DBH Class	1	2	3	4	5	Lower Volume by
	Volume in board feet					
10	30	50	80	110		1.5%
12	40	70	120	180	240	8.0
14	60	90	170	240	310	12.0
16	80	120	220	310	380	15.5
18	110	150	270	390	470	18.5
20	160	180	330	460	610	21.0
22		220	390	590	760	23.0

Example:
From Table IV the volume of a 22 inch, 4 log tree = 590 board feet.
From Table IV, col. 7 this volume must be lowered by 23%.
0.23 x 590 = 135.7 bd. ft. 590 - 136 = 454, or 450 Dec. C.
This volume (450) is then entered in its proper place in Table V.

Table V

Corrected volume table

DBH Class	Number of 16 foot logs 1	2	3	4	5
	Volume - board feet				
10	30	50	80	110	
12	40	60	110	170	220
14	50	80	150	210	270
16	70	100	190	260	320
18	90	120	220	320	380
20	130	140	260	380	460
22		170	300	450	590

Aggregate Difference = + 1.9%
Average Individual Deviation = ± 15.2%
} measures of error of revised table.

should be the minimum. The sizes of the trees measured locally should be well distributed over the range of sizes included in the volume table to be checked.

A further criterion of whether sufficient trees have been measured for an adequate check can be obtained by inspection of the plotted points of Fig. 2 and the magnitude of the final average individual deviation.

After the volume table has been checked and corrected for local conditions there remains the problem of revising it so that volumes for trees of a given d.b.h. and site class can be determined. For this purpose the volume table can be transferred into graphic form as shown by the broken lines of Fig. 3. In doing this it must be remembered that the volume for a two log tree must be plotted opposite a merchantable height on the vertical ordinate of the graph paper equal to two 16 foot sections plus a trimming allowance 0.3 feet per log. Volumes for the various log heights must be plotted opposite their corresponding merchantable heights. Such heights are:

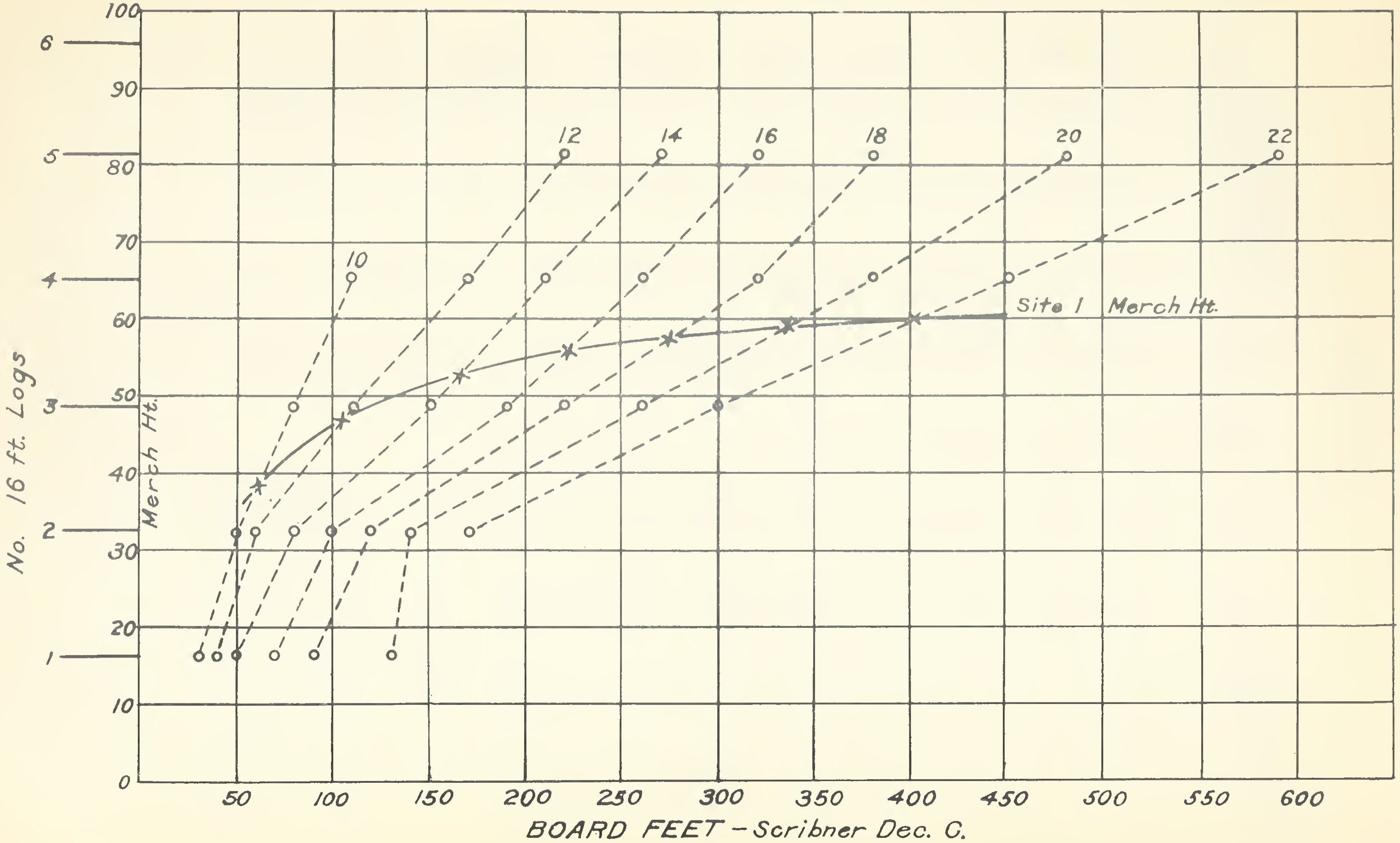
No. logs	Merchantable length.
1	16.3
2	32.6
3	48.9
4	65.2
5	81.5
6	97.8

For each site cl ss a curve of merchantable height over d.b.h. can then be constructed for each species from sample tree data as shown in Fig. 1. The smoothed values from this curve can then be transferred to Fig. 3, plotting on each d.b.h. class curve the corresponding merchantable length of stem. A new volume table showing tree volumes by d.b.h. and site class can then be read from Fig. 3.

It is possible of course to construct a base table such as Table V for each site class. This requires only that the local trees used in the check be segregated by site class. All trees measured on Site I would then be used to revise the original table to get a new Site I volume table and likewise with Sites II and III. However, it is doubtful if a sufficient number of trees in all three sites can be collected so that the three separate site class volume tables can be built. From 2 to 3 months work on 2 national forests of Region 8 have shown that the majority of the trees measured on logging jobs come from Site II and that the number of trees from Sites I and III are insufficient for individual volume table checks. Consequently, for the present job it is recommended that all trees of a single species be measured on local logging jobs and thrown together regardless of site for the purpose of ehcking and revising regional volume tables for local use. This revised volume table can then be used as the basis for plotting such data as Fig. 3. One of the most important effects of site on tree volume will then be provided for by the merchantable height curves superimposed on such curves as Figure 3. However, a site class should be assigned to each tree scaled for the purpose of checking volume tables. In the course of time enough data will accumulate so that an investigation can be made of the effect of site on the volumes of trees of the same dimensions, and if found to be significant new base tables by site classes can be made for future use.

Volume tables derived from the foregoing procedure give gross volumes when applied to any given area of forest land. These gross volumes must be

FIG. 3



10
12
14
16
18
20
22
Site 1 Merch Ht.
Merch Ht.
No. 16 ft. Logs
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
90
100
50
100
150
200
250
300
350
400
450
500
550
600
BOARD FEET - Scribner Dec. C.

Final check of Revised Volume Table

Table VI
Yellow Poplar

1	2	3	4	5	6	7
			Volume in bd.ft.			
Tree No.	DBH In.	No. logs	from corrected table	local trees	Diff.	Diff in %
1	10.0	2	50	60	10	20.0
2	10.4	2 1/2	69	70	1	1.4
3	10.8	3 1/4	102	90	12	11.8
4	11.0	3	95	110	15	15.8
5	11.0	2 3/4	85	70	15	17.6
6	11.0	3	95	90	5	5.3
7	11.2	2 7/8	93	80	13	14.0
8	11.7	2	58	70	12	20.7
9	11.8	3 1/4	121	180	59	48.8
10	11.9	2	60	70	10	16.7
11	12.0	2 1/2	85	80	5	5.9
12	12.1	2 5/8	93	100	7	7.5
13	12.2	3 1/4	129	110	19	14.7
14	12.4	2	64	100	36	56.2
15	12.4	3 5/8	156	140	16	10.2
16.	12.6	3 7/8	174	140	34	19.5
17	12.6	3 1/2	152	140	12	7.9
18	12.7	2 1/2	81	100	19	23.4
19	12.8	2 1/4	82	75	7	8.5
20	13.0	3 1/2	160	130	30	18.8
21	13.0	3 1/2	160	140	20	12.5
22	13.0	3 5/8	168	150	18	10.7
23	13.2	4 1/8	201	210	9	4.5
24	13.2	3	134	120	14	10.4
25	13.3	3 1/4	151	120	31	20.5
26	13.4	2 5/8	114	120	6	5.6
27	13.6	3 5/8	180	180	0	0.0
28	13.6	3 7/8	194	190	4	2.1
29	13.7	3 3/8	166	150	16	9.6
30	13.7	3	144	110	34	23.6
31	13.8	2 1/4	95	130	35	36.8
32	14.0	2 3/4	132	110	22	16.7
33	14.2	3	154	140	14	9.1
34	14.4	3 1/8	166	180	14	8.4
35	14.4	4	220	220	0	0.0
36	14.6	4 1/8	232	210	22	9.5
37	14.6	4	225	160	65	28.9
38	14.7	3 7/8	220	360	140	63.6
39	14.8	3 1/4	184	160	24	13.0
40	14.9	4 1/8	240	180	60	25.0
41	15.0	3 3/4	219	240	21	9.6
42	15.0	2 5/8	140	160	20	14.3
43	15.1	3 1/2	205	190	15	7.3
44	15.2	4 1/4	255	240	15	5.9
45	15.4	3 5/8	220	200	20	9.1
46	15.4	3 3/8	203	200	3	1.5
47	15.6	3 3/4	226	240	14	6.2
48	15.6	4 3/8	266	260	6	2.3
49	15.6	2 7/8	171	220	49	28.6
50	15.7	2 3/8	130	120	10	7.7
51	15.9	3 1/4	205	270	65	31.7
52	16.6	3 7/8	267	230	37	13.9
53	16.6	2 3/8	141	170	29	20.6
54	16.6	4 3/8	300	340	40	13.3
55	16.7	3 3/4	261	260	1	.4
56	16.7	3	200	180	20	10.0
57	17.1	4 1/4	309	290	19	6.2
58	17.1	3	206	270	64	31.0
59	18.6	3 3/4	312	310	2	.6
60	18.7	3 1/4	261	260	1	.4
61	19.0	4 3/4	410	240	170	41.5
62	19.1	4 1/4	374	280	94	25.1
63	21.3	3	286	330	44	15.4
			11051	10845		957.8
			- 10845			
			-206			

$$\frac{20600}{11051} = + 1.9\% \text{ Aggregate Difference}$$

$$\frac{957.8}{63} = \pm 15.2 \text{ Average Individual Deviation}$$